

**The Three P's of  
Total Risk Management**

Andrew W. Lo

MIT Sloan School of Management

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# Outline

1. Perfect Market Timing
2. Loss Aversion
3. Principles of Statistical Risk Management
4. Risk vs. Uncertainty
5. Preferences Under Certainty
6. Preferences Under Uncertainty
7. Eliciting Preferences
8. Limitations of Expected Utility
9. Interpreting Probabilities
10. Summary and Conclusions
11. Additional References

## Perfect Market Timing

### Monthly Returns 1926:1–1998:12

| Statistic    | S&P 500 | T-Bills |
|--------------|---------|---------|
| Mean         | 1.05%   | 0.31%   |
| Std. Dev.    | 5.67%   | 0.26%   |
| Minimum      | −29.73% | −0.06%  |
| Median       | 1.34%   | 0.28%   |
| Maximum      | 42.56%  | 1.35%   |
| Sharpe Ratio | 0.45    | 0.00    |
| Total Return | \$2351  | \$15    |

What About Perfect Market Timing?

- Suppose  $R_t = \text{Max} [ \text{S\&P 500} , \text{T-Bills} ]$
- What Does \$1 Grow To In This Case?

## Loss Aversion

Choose A or B:

A: Sure Gain of \$240,000

B:  $\begin{cases} \$1,000,000 & \text{with probability } 25\% \\ \$0 & \text{with probability } 75\% \end{cases}$

Choose C or D:

C: Sure Loss of \$750,000

D:  $\begin{cases} \$0 & \text{with probability } 25\% \\ -\$1,000,000 & \text{with probability } 75\% \end{cases}$

Equivalent Choices:

A+D:  $\begin{cases} \$240,000 & \text{with probability } 25\% \\ -\$760,000 & \text{with probability } 75\% \end{cases}$

B+C:  $\begin{cases} \$250,000 & \text{with probability } 25\% \\ -\$750,000 & \text{with probability } 75\% \end{cases}$

Financial Technology: Framework for Analysis

# Principles of Statistical Risk Management

The Risk Management Process:

- Identify Risk Exposures
- Evaluate Value-At-Risk (VAR)
- Target Risks To Hedge
- Select Hedging Vehicles
- Evaluate Post-Hedge VAR

But How Much Risk Should Be Hedged?

- Individual vs. Corporate Preferences
- M&M Irrelevancy Applied To Risk
- Frictions Motivate Risk Management

The Economist's Answer:

It Depends!

$$E \left[ (1 + R_{it}) \frac{\rho U'(C_t)}{U'(C_{t-1})} \right] = E \left[ (1 + R_{jt}) \frac{\rho U'(C_t)}{U'(C_{t-1})} \right] = 1$$

## Risk vs. Uncertainty

Consider Urn A With 100 Red and Black Balls

- 50 Red, 50 Black
- Consider Drawing One Ball
- Bet On Color, \$10,000 Payoff
- Which Color Would You Prefer?
- How Much Would You Pay?

Consider Urn B With 100 Red and Black Balls

- Proportion Unknown
- Consider Drawing One Ball
- Bet On Color, \$10,000 Payoff
- Which Color Would You Prefer?
- How Much Would You Pay?

Suppose You Have To Bet—Which Urn Would You Prefer?

- Statistical Aspect (Probabilities)
- Economic Aspect (Prices)
- Personal Aspect (Preferences)

Risk Management Must Integrate These Three Aspects

# Preferences Under Certainty

Notion of Utility:

- Index of Happiness
- Quantitative Analysis
- Cardinal vs. Ordinal Measures
- Foundation of Microeconomics

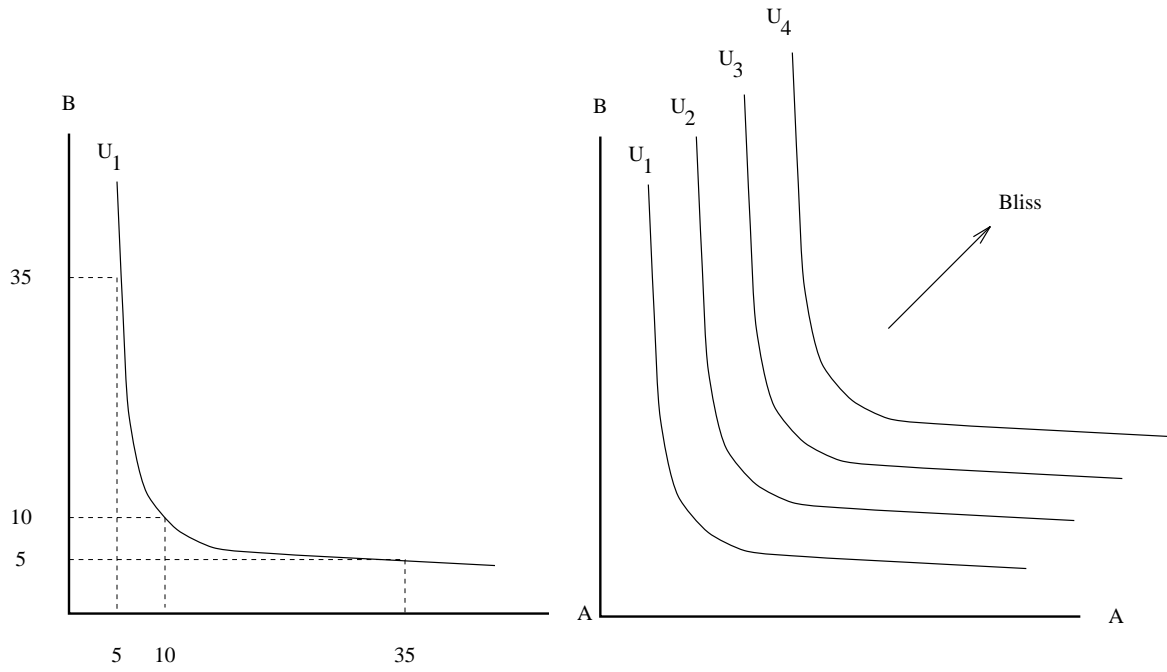
Suppose:

- For Any Two Goods  $A$  and  $B$  (Completeness):

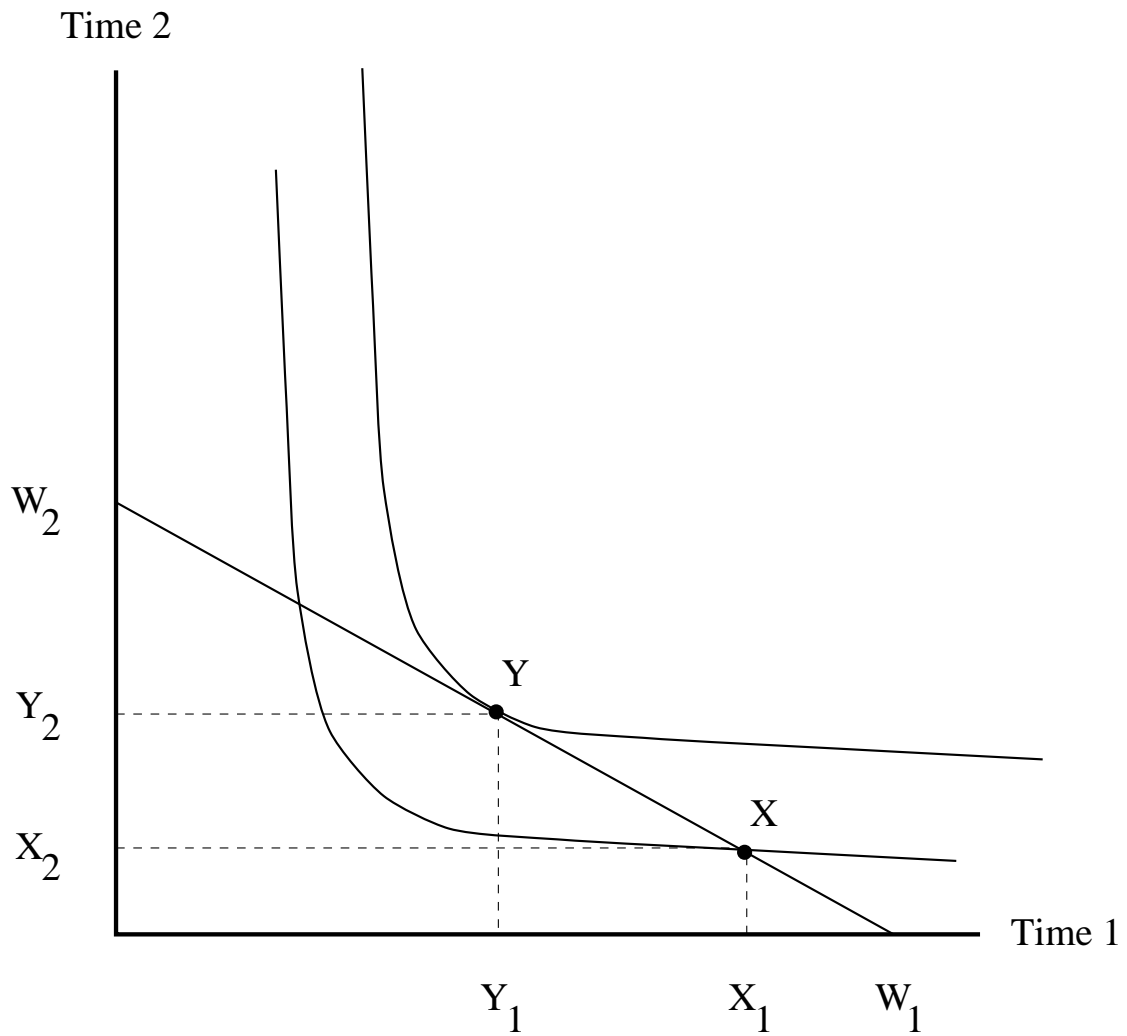
$$A \succ B \text{ or } A \prec B \text{ or } A \sim B \quad (1)$$

- $A \succ B \succ C \Rightarrow A \succ C$  (Transitivity)
- More Is Preferred To Less (Non-Satiation)
- “Diminishing Marginal Utility”

Implies Ordinal Utility Theory



Suppose Capital Market Exists:



$Y \succ X$ ,  $Y$  Maximizes Utility

$$W_1 = X_1 + \frac{1}{1+R}X_2 \quad (2)$$

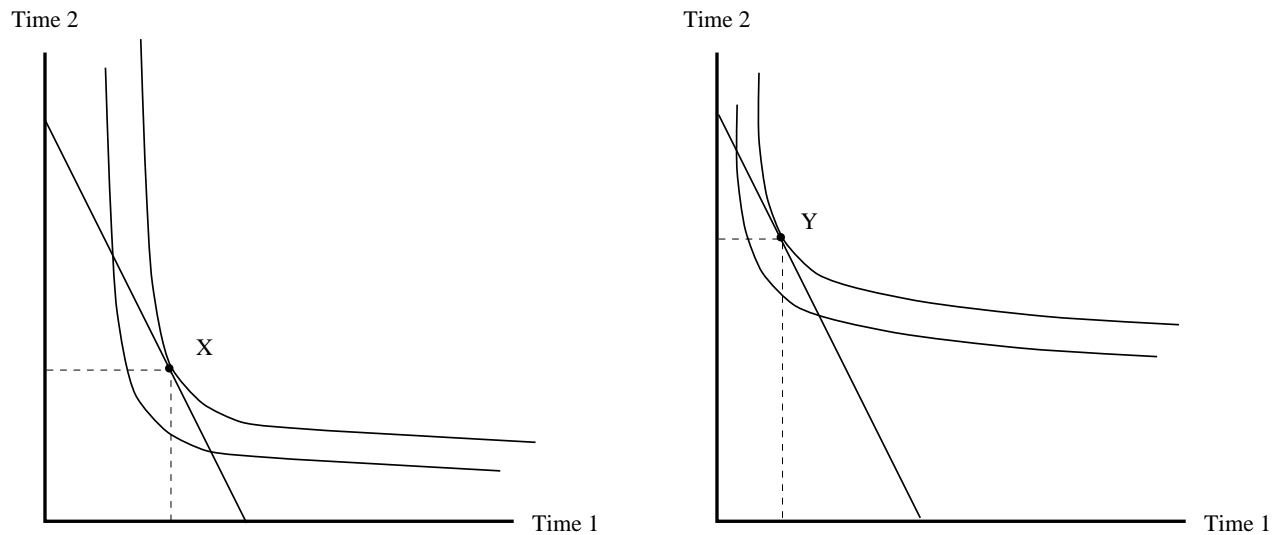
$$W_2 = (1+R)W_1 = (1+R)X_1 + X_2 \quad (3)$$

$$C_2 = -(1+R)C_1 + k \quad (4)$$

$$k \equiv (1+r)W_1 \quad (5)$$

⇒ Budget Constraint

$$W_1 = C_1 + \frac{1}{1+R}C_2 \quad (6)$$



⇒ Capital Markets Allows Temporal Shift of Cash

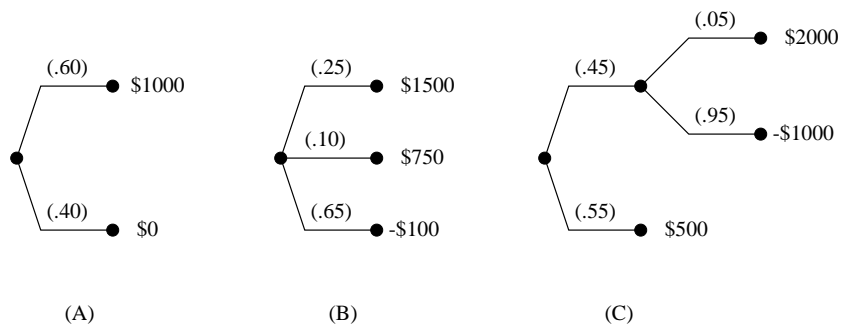
# Preferences Under Uncertainty

Under Certainty, Finance Theory Complete

- Present Values, Future Values, NPV
- Pricing Models Trivial
- Basic Microeconomics

But What About Uncertainty?

- $W$  Is A Random Variable
- $U(W)$  Is Also Random
- Preferences For Gambles
- Maximize Expected Utility  $E[U(W)]$



Suppose:

- Indifference Between Compound and One-Stage Gambles
- Completeness of Preferences
- Negative Transitivity
- Continuity
- Substitution

Then Expected Utility Theory Holds:

- Preferences Can Be Captured By  $E[U(W)]$
- $G_1 \succ G_2 \Leftrightarrow E[U(G_1)] > E[U(G_2)]$
- Unique Only Up To Linear Transformation
- How To Find  $U(W)$ ?

# Eliciting Preferences

Thought Experiment:

- Consider Gaining \$50,000 vs. Losing \$10,000

$$U(\$50,000) = 1 \quad (7)$$

$$U(-\$10,000) = 0 \quad (8)$$

- Let  $G_1$  Be 50/50 Gamble

$$G_1 = \begin{cases} \$50,000 & \text{with probability } .5 \\ -\$10,000 & \text{with probability } .5 \end{cases} \quad (9)$$

- Find Certainty Equivalent  $X_1$  of  $G_1$

$$U(X_1) = E[U(G_1)] = .5 \times 0 + .5 \times 1 = .5 \quad (10)$$

- Define  $G_2$  and  $G_3$  Similarly

$$G_2 = \begin{cases} \$50,000 & \text{with probability } .5 \\ X_1 & \text{with probability } .5 \end{cases} \quad (11)$$

$$G_3 = \begin{cases} -\$10,000 & \text{with probability } .5 \\ X_1 & \text{with probability } .5 \end{cases} \quad (12)$$

- This Yields Five Points

$$U(-\$10,000) = 0$$

$$U(X_2) = .75$$

$$U(X_1) = .50$$

$$U(X_3) = .25$$

$$U(\$50,000) = 1$$

- Interpolate To Get  $U(\cdot)$

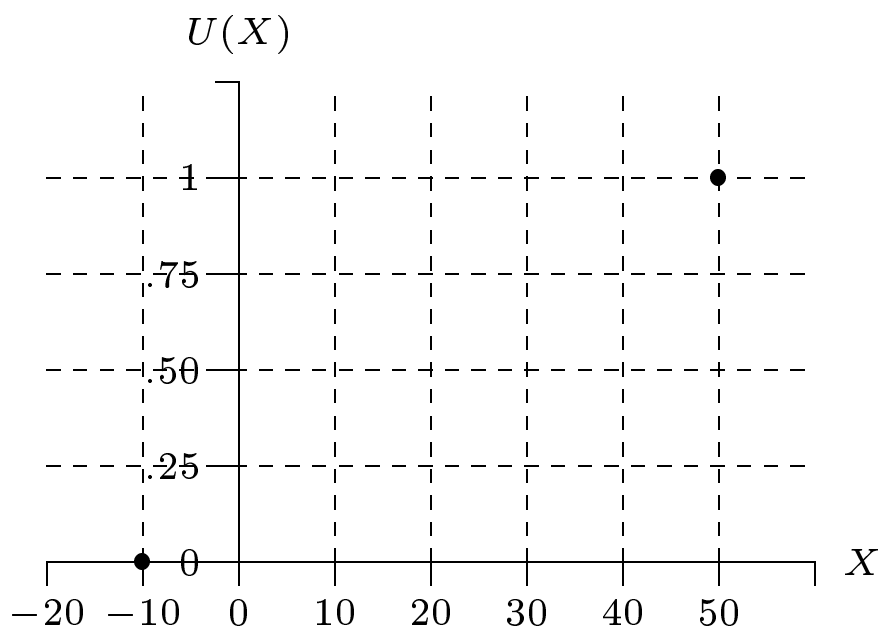


Figure 1: Your Utility Function

# Limitations of Expected Utility

Mean-Variance Preferences:

- Investors Like Mean, Dislike Variance (CAPM)
- $U(\mu, \sigma^2) = a\mu - b\sigma^2, a > 0, b > 0$
- Consistent With Expected Utility?
- Consider Second-Order Taylor Expansion

Violations of Expected Utility:

- Framing and Context Effects (Kahneman and Tversky)
- Allais Paradox
- Ellsberg Paradox

What About Intertemporal Trade-Offs?

- Risks Today vs. Expected Returns Next Year
- Expected Returns Today vs. Risks Next Year
- Diversification Over Time?

# Interpreting Probabilities

## VAR Based On Probabilities (RiskMetrics):

Value at Risk is an estimate, with a predefined confidence interval, of how much one can lose from holding a position over a set horizon. Potential horizons may be one day for typical trading activities or a month or longer for portfolio management. The methods described in our documentation use historical returns to forecast volatilities and correlations that are then used to estimate the market risk. These statistics can be applied across a set of asset classes covering products used by financial institutions, corporations, and institutional investors.

- Probability of Extreme Dollar Losses
- Conditional or Unconditional?
- Consistency Across Portfolios?
- Consistency Across Time?
- Prior Information?
- Preferences?

## Example: AIDS Testing:

- Suppose AIDS Test is 99% Accurate:

$$\begin{aligned}\text{Prob}(+|\text{AIDS}) &= 0.99 \\ \text{Prob}(-|\text{No AIDS}) &= 0.99\end{aligned}$$

- Suppose You Test Positive!
- What's Your Probability of AIDS???

Baye's Rule:

$$\text{Prob}(\text{AIDS}|+) = \text{Prob}(+|\text{AIDS}) \times \frac{\text{Prob}(\text{AIDS})}{\text{Prob}(+)} \quad (13)$$

$$\begin{aligned} \text{Prob}(+) &= \text{Prob}(+|\text{AIDS}) \times \text{Prob}(\text{AIDS}) + \\ &\quad \text{Prob}(+|\text{No AIDS}) \times [1 - \text{Prob}(\text{AIDS})] \end{aligned} \quad (14)$$

$$\text{Prob}(\text{AIDS}) \approx \frac{250K}{250M} = 0.001 \quad (15)$$

What's Your Estimate of Prob(AIDS|+)??

$$\text{Prob}(\text{AIDS}|+) = \text{Prob}(+|\text{AIDS}) \times \frac{\text{Prob}(\text{AIDS})}{\text{Prob}(+)}$$

$$= 0.99 \times \frac{0.001}{\text{Prob}(+)}$$

$$\begin{aligned} \text{Prob}(+) &= \text{Prob}(+|\text{AIDS}) \times \text{Prob}(\text{AIDS}) + \\ &\quad \text{Prob}(+|\text{No AIDS}) \times [1 - \text{Prob}(\text{AIDS})] \\ &= 0.99 \times 0.001 + 0.01 \times [1 - 0.001] \\ &= 0.01098 \end{aligned}$$

$$\text{Prob}(\text{AIDS}|+) = 0.99 \times \frac{0.001}{0.01098} = 0.090164$$

Conditioning On Prior Information Is Critical:

- Part of Prior Information Is Preferences
- Incentives
- Experience, Judgment, Intuition
- Biology

Significance of Zero-Probability Events:

- Suppose Event  $E$  Has Never Happened
- Tendency To Assume  $\text{Prob}(E) = 0$
- If Others Think  $\text{Prob}(E) \neq 0 \Rightarrow$  “Arbitrage”
- But This Can Lead To Huge Losses
- Objective vs. Subjective Probabilities

Emerging Field  $\Rightarrow$  Financial Decision Analysis

# Summary and Conclusions

## Statistical Vs. Economic Risk Management

- Economic Risks Vs. Statistical Risks
- S-VAR: Probabilities
- E-VAR: Probabilities, Preferences, Prices

## Preferences Are Central:

- Whose Preferences?
- Behavioral Biases Exist
- Useful For Survival In Early Civilization
- Dangerous For Survival In Modern Civilization
- Know Thyself!
- Three P's of Total Risk Management

## Economic Risk Management More Challenging

- Requires Additional Structure
- Requires More Sophisticated Estimation and Inference
- Requires Careful Interpretation
- More Relevant For Management of Risks

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